In other words, instead of low frequency periodics being a slow moving offset in the center of travel of the inertial mass with the higher frequency periodics being damped at the extremes of travel, as in the addition method, a gain factor approach of the sum of products method keeps the inertial mass near its "sweet spot" in the center of range of travel by modulating the peak amplitude about the center of range of travel, rather than displacing the center of range of travel.

[0109] The right lobe in waveform **342** is lower than the left lobe in graph **340**. This is because a small portion of the low frequency signal is passed by the high pass filter and appears as part of the signal in the output waveform. Likewise, a small portion of the high frequency signal contributes to the modulating envelope as well.

[0110] Normalization

[0111] It should be noted that, in the waveform **342**, the amplitude of the waveform exceeds the full scale values of +1 and −1, which would correspond to a 100% duty cycle. As with the addition method described previously, the waveform may be clipped, resulting in signal distortion.

[0112] To reduce the clipping effect, a normalized filter can be used. The normalized filter works similarly to the standard envelope filter, but adds a normalization term $1/(1-\beta_i)$ that constrains each of the effects to a number in the range of −1 to +1. As the envelopes are calculated, the functions are likewise normalized, and frequency, amplitude and saturation scaling is applied at the last step of the process before the effect values are re-scaled into amplifier magnitude values, such as PWM values.

[0113] The waveform is normalized in that although both waveforms are of maximum magnitude, the combined effect is of magnitude 1, not magnitude 2. In fact, if two waveforms of magnitude 0.5 were combined, the output waveform would have a peak output of 1. Only if the sum of the effect magnitudes is less than 1 will the combined effect be below 1, and will simply be the sum of the composite waveform magnitudes. The normalization can be thought of as an automatic gain control that scales the output of the filter so that the peak output function will never exceed the maximum commandable amplitude.

[0114] To normalize the effects, the following equations can be used:

$$\alpha_i=[1-1/(1+f_i/f_c)] \tag{6}$$

$$\beta_i=1/(1+f_i/f_c) \tag{7}$$

$$\alpha_{i\ norm}=\alpha_i/(\Sigma_{j-1,n}\alpha_j) \tag{8}$$

$$\beta_{i\ norm}=\beta_i/(\Sigma_{j-1,n;j}\beta_j) \tag{9}$$

[0115] Once the normalized gain terms $\alpha_{i\ norm}$ and $\beta_{i\ norm}$ are found in the equations above, the equations (4) and (5) can be used to find the resulting waveform to be output. The central idea is to modify the low and high pass gain terms so that the sum of the low pass gains equals one, and, likewise, the sum of the high pass gains equals one.

[0116] The normalized output from the filter of the present invention is shown as waveform **352** in graph **350** of **FIG. 12**. The amplitude of the waveforms does not exceed +1 and −1. In other embodiments, as mentioned above, a small amount of excess over the saturation limits can be permitted, since the minor amount of such clipping may not be noticeable to the user.

[0117] Simplified Sum of Products

[0118] The pure solution

$$f_{composite}(t)=\Sigma_i f_i(t)=\alpha_i M_i \sin(\omega_i t+\phi_i)[|\Sigma_{j,j}\beta_i \sin(\omega_j t+\phi_j)|]$$

[0119] can be simplified as

$$f_{composite}(t)=\Sigma_i f_i(t)=\alpha_i M_i \sin(\omega_i t+\phi_i)[|\Sigma_j\beta_i \sin(\omega_j t+\phi_j)|].$$

[0120] Note that a single envelope function is applied to each source effect. It is not necessary to calculate a separate envelope function for each source effect. This reduces the number of function value computations from n*(n−1) to n+1. Although each n source effect makes a contribution to its own envelope function, the contribution will distort the envelope function to a small degree as high frequency source effects have small β_n values and contribute little to the envelope function. The simplified solution may be more appropriate in some embodiments, e.g. a less sophisticated processor or as a firmware solution as explained below.

[0121] Normalization and Combining

[0122] The waveform **352** resulting from the normalizing filter described above would typically feel quite adequate to the user. However, the output can be improved further. In the graph **350** of **FIG. 12**, the output is shown to go to zero at 0.5 seconds. Waveform **350** conveys both the high frequency signal information as well as the low frequency envelope detail, but the amplitude of the resulting waveform **352** may not be desirable to go to zero. For example, a constant high frequency repeating pulse may be desired during the output of the low frequency, and the user would notice when the waveform went to zero. To alleviate this zeroed portion of the waveform, a superposition of the envelope with the high frequency signal is performed to convey the low frequency information and the high passed high frequency signal. This is accomplished by a normalizing and combining filter. The combining filter combines the envelope modulated low frequency information with the high passed effect signal.

[0123] The normalized and combined filter can be implemented with the following equations:

$$\alpha_i=[1-1/(1+f_i/f_c)] \tag{10}$$

$$\beta_i=1/(1+f_i/f_c) \tag{11}$$

$$A_{i\ norm}=\alpha_i M_i/(\Sigma_{j-1,n}\alpha_j M_j) \tag{12}$$

$$B_{i\ norm}=\beta_i M_i/(\Sigma_{j-1,n}\beta_j M_j) \tag{13}$$

$$G=G_{sat} \text{ if } \Sigma M_i>G_{sat} \text{ else } G=\Sigma M_i;\ G_{sat}=1 \text{ for normalized} \tag{14}$$

$$f_{combined}(t)=G\Sigma_i\{A_i M_i \sin(\omega_i t+\phi_i)[1+\Sigma_{j,j}|B_i \sin(\omega_j t+\phi_j)|/(1-B_i)]\} \tag{15}$$

[0124] G is a gain term that can be used to adjust the amplitude of the final waveform, where G_{sat} is a predetermined saturation level above which the amplitude cannot go. The G_{sat} term is typically 1 when normalized, but can be increased (e.g., to 1.2) if some saturation is desired. The combining equation (15) adds the high frequency signal to the normalized waveform result shown in **FIG. 12**. **FIG. 13** is a graph **360** showing a waveform **362** that is the final result of the normalizing and combining filter. The waveform conveys both the low frequency periodic and the constant high frequency periodic. As shown, the portion of the waveform **362** around t=0.5 seconds has an amplitude substantially greater than zero, so that the user never feels the absence of the high frequency waveform with the low frequency envelope.